

Exercise 56

Make a rough sketch of the curve $y = x^n$ (n an integer) for the following five cases:

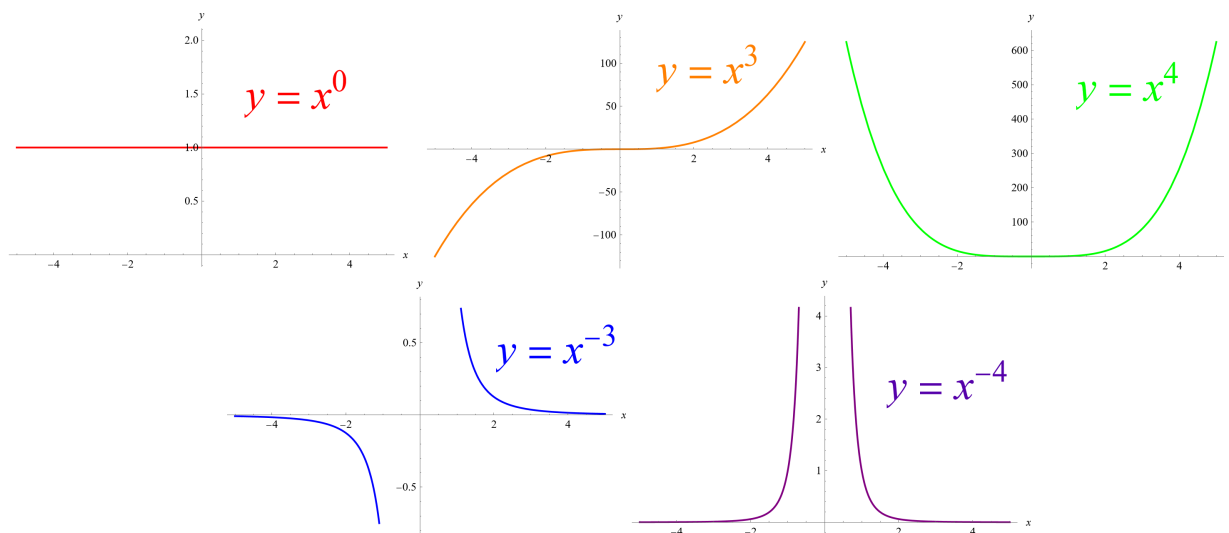
- (i) $n = 0$ (ii) $n > 0, n$ odd
 (iii) $n > 0, n$ even (iv) $n < 0, n$ odd
 (v) $n < 0, n$ even

Then use these sketches to find the following limits.

- (a) $\lim_{x \rightarrow 0^+} x^n$ (b) $\lim_{x \rightarrow 0^-} x^n$
 (c) $\lim_{x \rightarrow \infty} x^n$ (d) $\lim_{x \rightarrow -\infty} x^n$

Solution

Below are some example graphs of $y = x^n$ versus x for various values of n .



Use them to determine the desired limits.

$$(a) \lim_{x \rightarrow 0^+} x^n = \begin{cases} \infty & \text{if } n < 0 \\ 1 & \text{if } n = 0 \\ 0 & \text{if } n > 0 \end{cases}$$

$$(b) \lim_{x \rightarrow 0^-} x^n = \begin{cases} -\infty & \text{if } n < 0 \text{ and } n \text{ is odd} \\ \infty & \text{if } n < 0 \text{ and } n \text{ is even} \\ 1 & \text{if } n = 0 \\ 0 & \text{if } n > 0 \end{cases}$$

$$(c) \lim_{x \rightarrow \infty} x^n = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n = 0 \\ \infty & \text{if } n > 0 \end{cases}$$

$$(d) \lim_{x \rightarrow -\infty} x^n = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n = 0 \\ -\infty & \text{if } n > 0 \text{ and } n \text{ is odd} \\ \infty & \text{if } n > 0 \text{ and } n \text{ is even} \end{cases}$$